RAN-2403081203022001

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M.Sc. (Sem.-III) Examination October - 2025

Statistics - Linear Models (302)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book
(2) Attempt all questions.
(3) Each question carries 14 marks.
(4) Answer any two out of the three bits given in each question.

Seat No.:

Student's Signature

Q.1. **a.** Consider the GMLM $(Y, X\beta, \sigma^2 I)$. Show that a linear parametric function $\lambda'\beta$ is estimable if and only if $\lambda' \in \mathscr{R}(X)$, the row space of X.

b. Consider the GMLM $(Y, X\beta, \sigma^2 I)$. Show that the normal equations $X'X\beta = X'Y$ are always consistent and if $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are any solutions of normal equations, then $X\widehat{\beta}_1 = X\widehat{\beta}_2$.

c. Two items A and B are weighed on a balance, first separately and then together to yield observations y_1, y_2, y_3. Find the least squares estimates of the true weights items A and B. Also find an unbiased estimator of σ^2.

Q.2. **a.** Let $X \sim N_n(\mu, V)$ with V nonsingular, A is real symmetric matrix of order n, show that
i. $E(X'AX) = tr(AV) + \mu'A\mu$
ii. $Cov(X'AX) = 2AV\mu$

b. Consider the model $y_1 = \beta_1 + \beta_2 + e_1$, $y_2 = \beta_2 + \beta_3 + e_2$, $y_3 = \beta_1 + \beta_3 + e_3$, where $E(e_i) = 0$, and $Var(e_i) = \sigma^2$, for i=1,2,3. Show that the parametric function $\lambda_1\beta_1 + \lambda_2\beta_2 + \lambda_3\beta_3$ is estimable if and only if $\lambda_1 = \lambda_2 + \lambda_3$.

c. i. Show that covariance between any linear function belonging to the error space and any BLUE is zero.
ii. What role does a function belonging to error space play? Explain.

Q.3. **a.** Consider the non-full rank linear model $Y = X\beta + \varepsilon$, where $\underline{\varepsilon} \sim N_n(0, \sigma^2 I)$. Let $H_0: K'\beta = \underline{m}$ be a testable hypothesis where K' is of type S×P and is of full row rank. Propose a suitable test criterion for testing H_0. Giving full justification, explain in detail how you will test the hypothesis H_0.

b. Let $X \sim N_n(0, V)$ with V known and non-singular. A is real symmetric idempotent matrix of order n. Show that $X'AX$ follows Chi-square distribution, if and only if AV is idempotent.

c. State and prove Cochran's theorem.

Q.4 **a.** Consider the GGMLM $(Y, X\beta, \sigma^2 V)$ of full rank with V known and nonsingular. Show that a necessary and sufficient condition for the ordinary least squares estimator of β to be best linear unbiased estimator is that $\psi(V^{-1}X) = \psi(X)$.

b. When $X \sim N_n(\mu, V)$ with V known and nonsingular, show that the quadratic forms $X'AX$ and $X'BX$ are independently distributed if and only if $AVB' = 0$.

c. Write a detail note on Canonical form of the linear hypothesis model $(Y, X\beta, \sigma^2 I)$ of rank r and Error and Estimation Spaces.

Q.5. **a.** Write a detail note on least square estimation with restrictions on parameter.

b. Write a detail note on independence of a linear form and a quadratic form.

c. Let $X \sim N_n(0,V)$ with V nonsingular. Derive the Moment Generating function of $X'AX$.